Twin Approaches to Structure Functions: Quark-Hadron Duality and the Resonance-Scaling Transition

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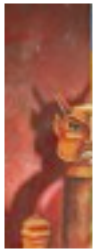
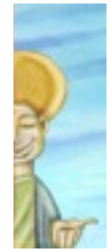
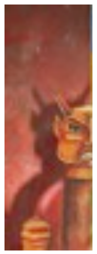
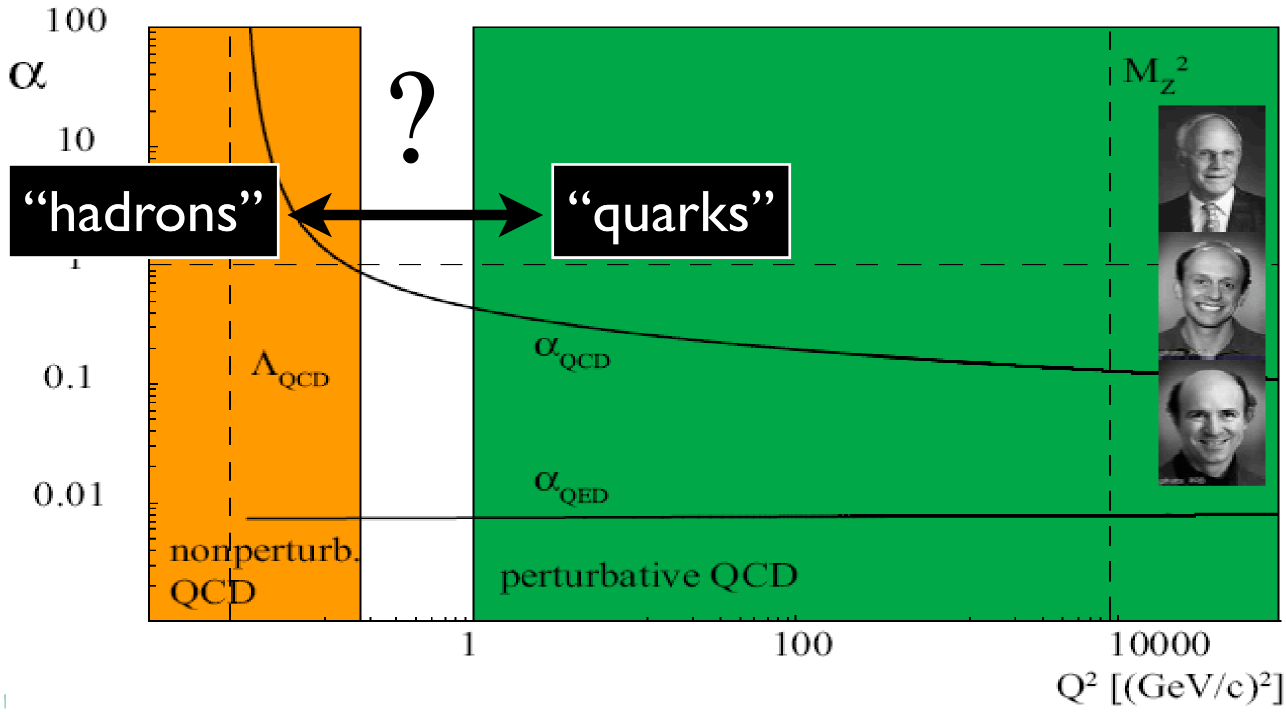


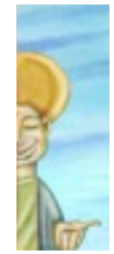
Outline



- Historical perspective
  - → examples from Nature
- Duality and QCD
  - → twists and moments
  - → nonperturbative models
- Implications for PDF analyses
- Outlook







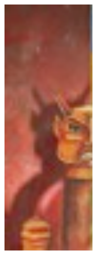
long distance

short distance

complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

→ can use either set of *complete* basis states to describe physical phenomena



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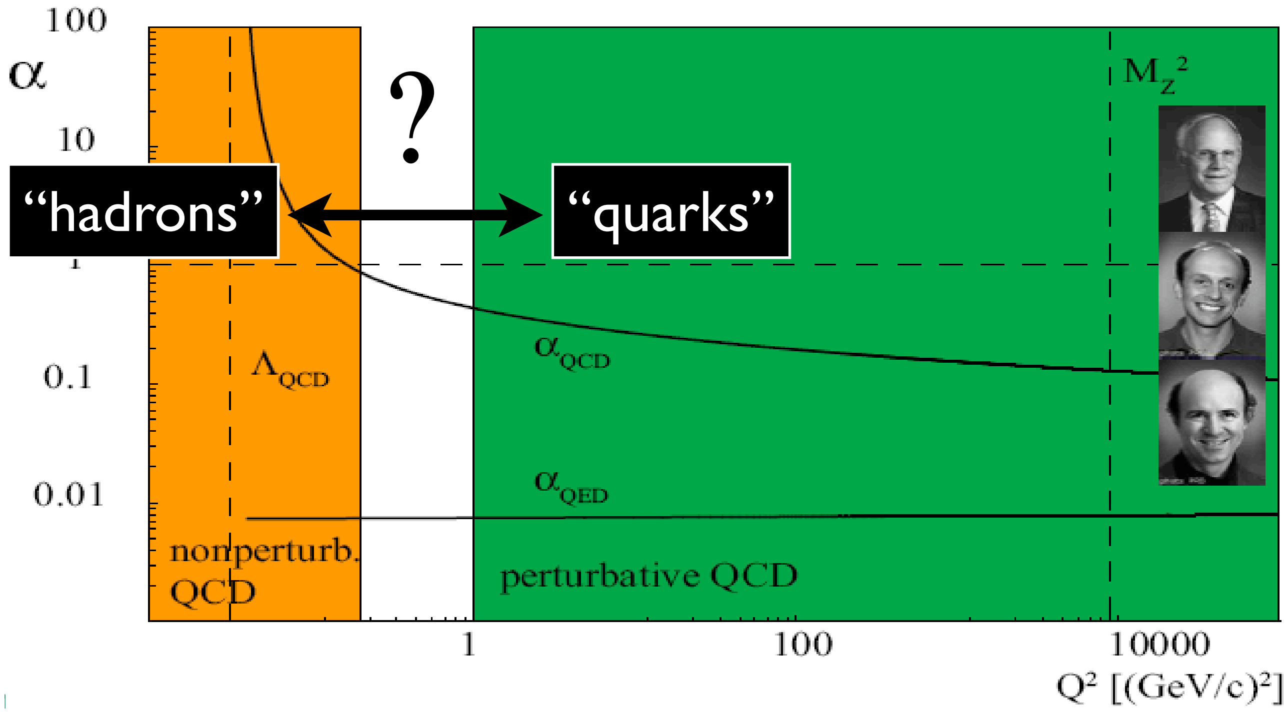

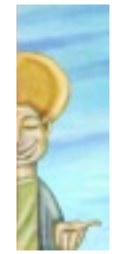

short distance

→ in practice, at finite energy typically have access only to *limited* set of basis states

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→ question is not *why* duality exists, but *how* it arises where it exists

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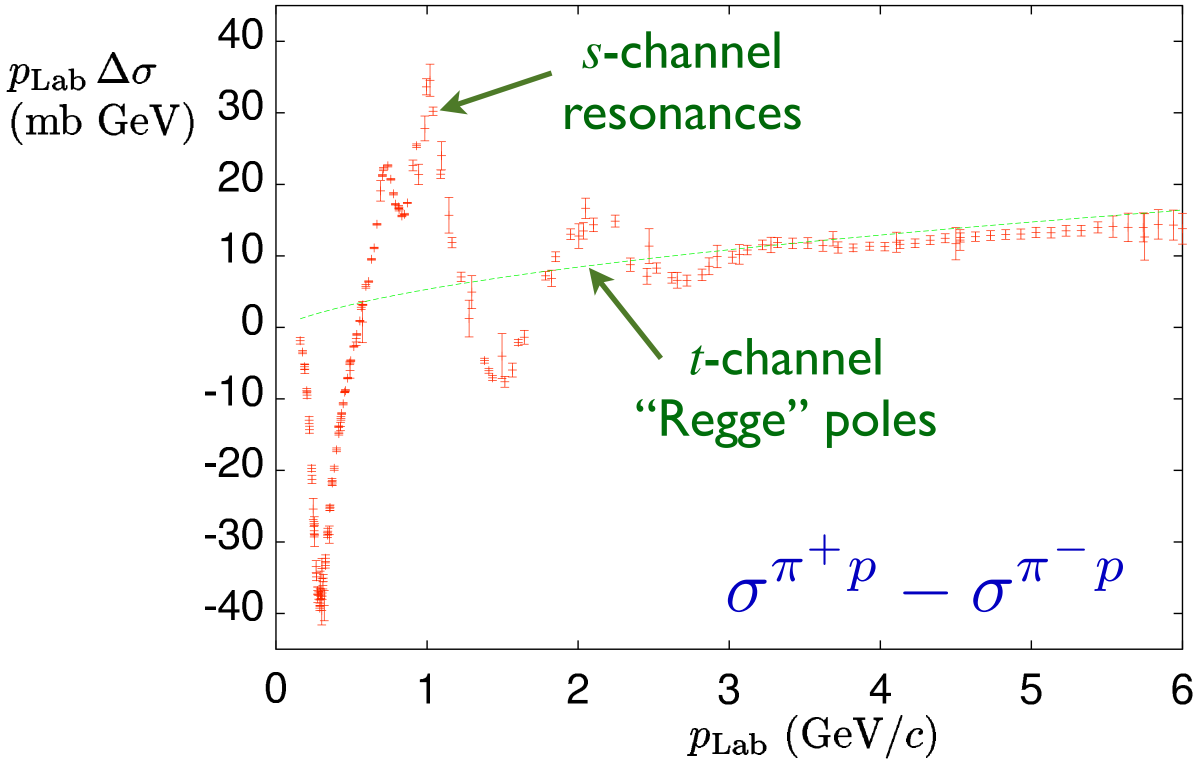
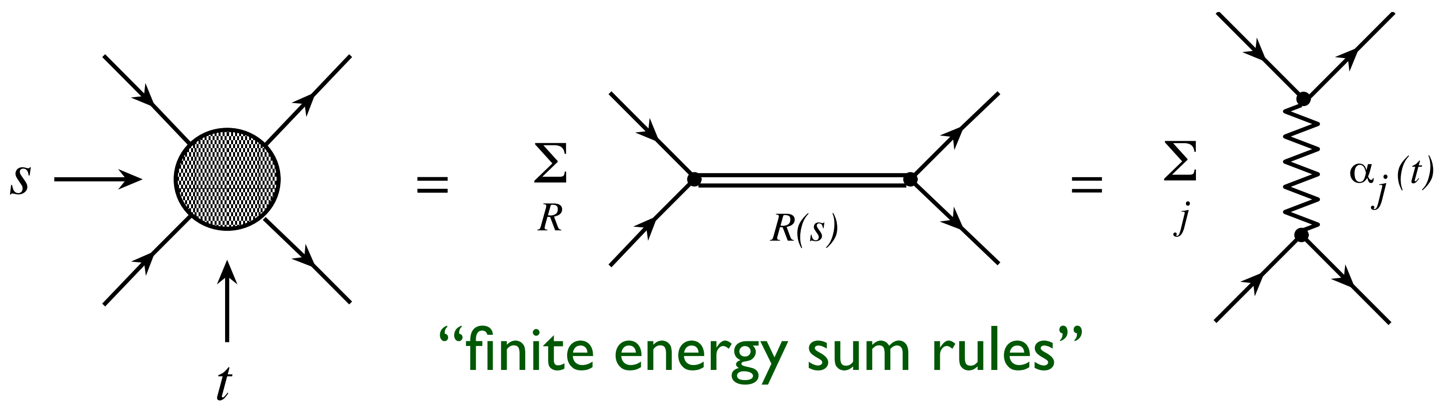
Duality in hadron-hadron scattering

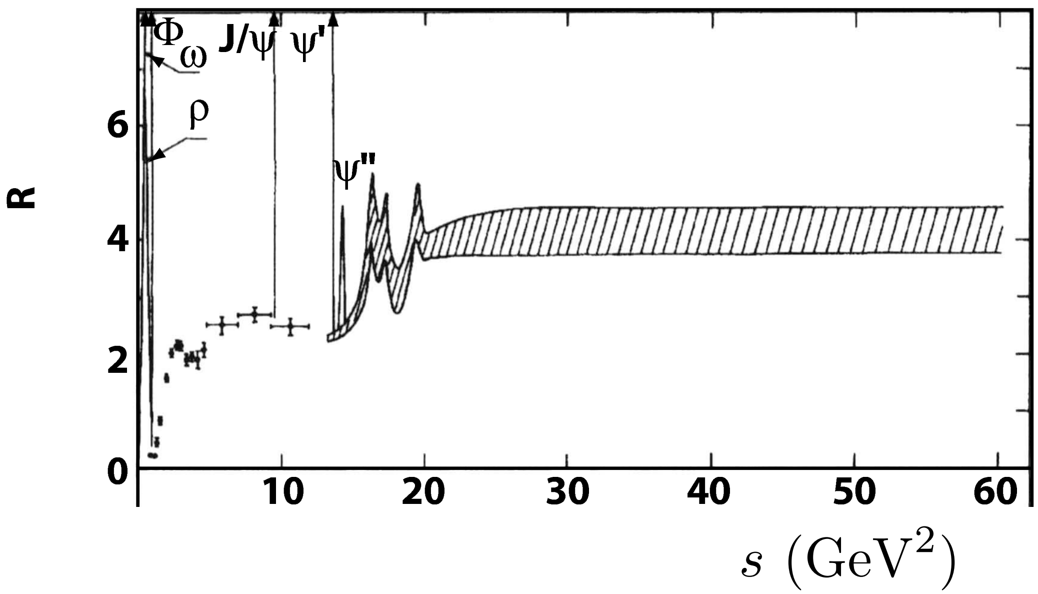




"finite energy sum rules"

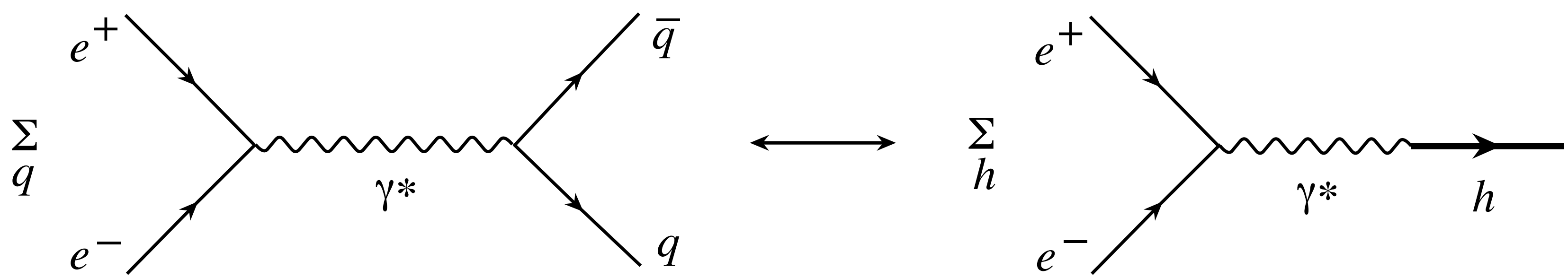
Igi (1962), Dolen, Horn, Schmidt (1968)

Duality in $e^+ e^-$ annihilation

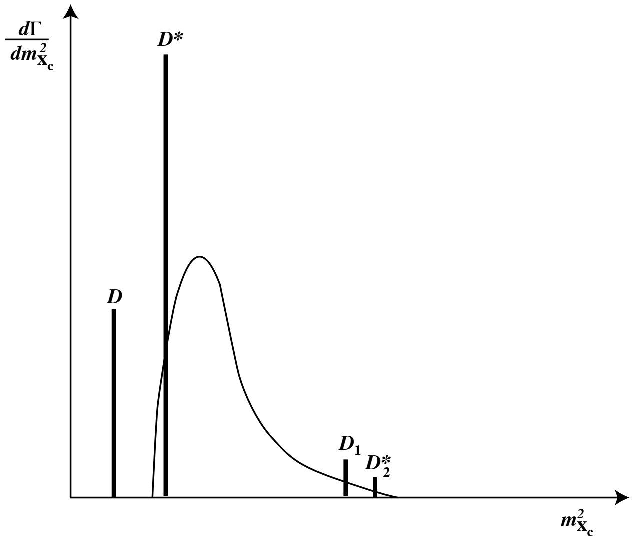


$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

→ total hadronic cross section at high energy averages resonance cross section



Duality in heavy meson decays



$$Q \to Q'\, l\, \bar{\nu}_l$$

$$\Gamma^q = \frac{G_F^2 \delta m^5}{15\pi^2} |V_{QQ'}|^2$$

$$(Q\bar{q}) \to X_{Q'}\, l\, \bar{\nu}_l$$

$$\Gamma^{\rm PS} = \frac{G_F^2 \delta m^5}{60\pi^2} |V_{QQ'}|^2$$

$$\Gamma^{\rm V} = \frac{G_F^2 \delta m^5}{20\pi^2} |V_{QQ'}|^2$$

$$m_Q + m_{Q'} \gg m_Q - m_{Q'} \gg \Lambda_{\rm QCD}$$

$$\delta m = m_Q - m_{Q'} \approx M_{(Q\bar{q})} - M_{Q'\bar{q}}$$

→ sum over hadronic-level decay rates
= quark-level decay rate

$$\Gamma^{\rm PS} + \Gamma^{\rm V} \longleftrightarrow \Gamma^q$$

*Voloshin, Shifman, SJNP **41**, 120 (1985); Isgur, PLB **448**, 111(1999)*

Duality in large-N_c limit

- 't Hooft model: QCD in $1+1$ dimensions in $N_c \to \infty$ limit

  → discrete spectrum of infinitely narrow $q\bar{q}$ bound states

  → Green's functions calculable *exactly*

- Structure function for n-th bound state

$$W_n \sim \sum_m \left| F^q_{nm}(Q^2) + F^{\bar{q}}_{nm}(Q^2) \right|^2 \delta(W^2 - M_n^2)$$

where form factors $F^q_{nm} \sim (-1)^m e_q m_q \, \phi_n / Q^2$
for quark distribution amplitude ϕ_n

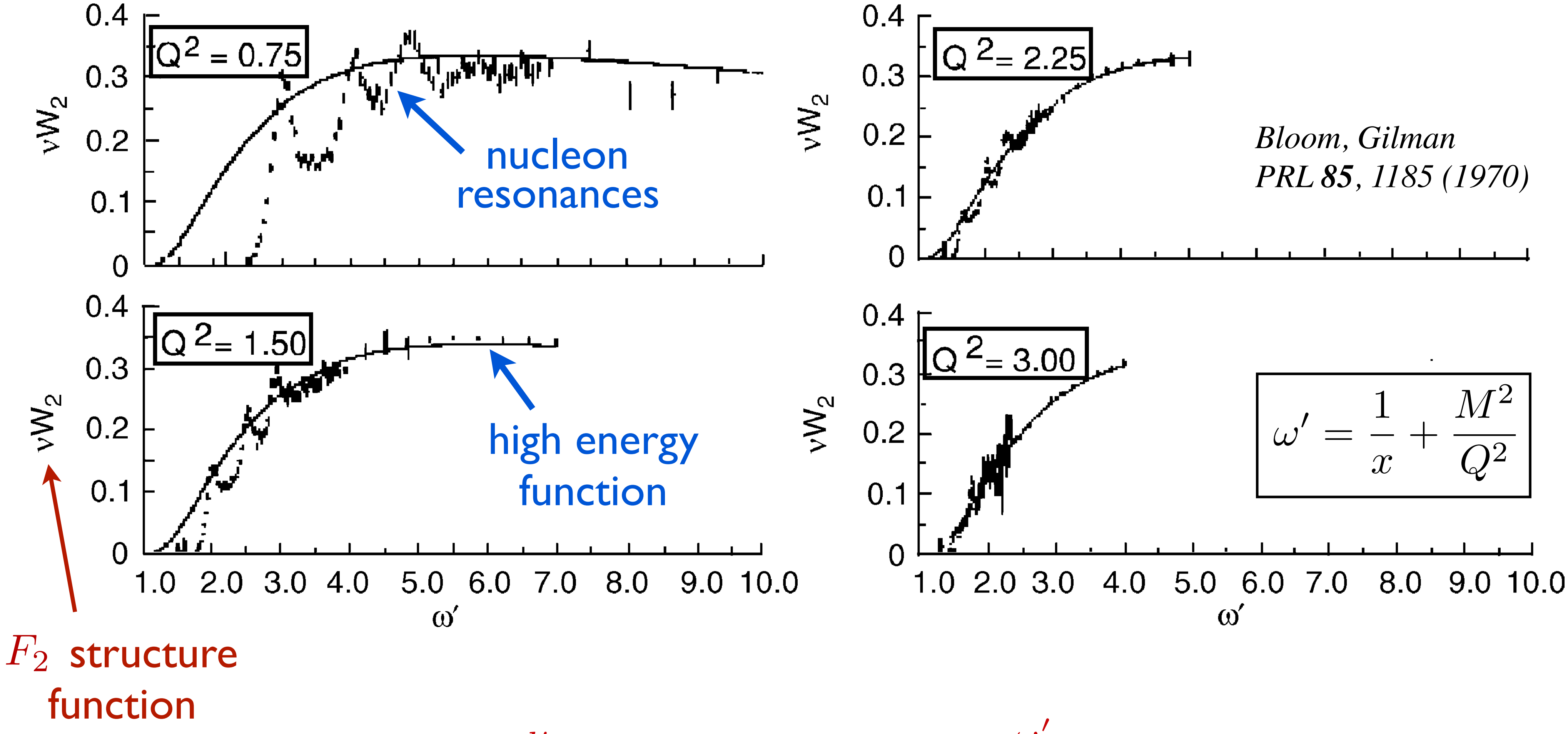
- Scaling structure function obtained in $Q^2 \to \infty$ limit
from δ-function spectrum

$$\nu^2 W_n \sim e_q^2 m_q^2 \, \phi_n^2(x) + e_{\bar{q}}^2 m_{\bar{q}}^2 \, \phi_n^2(1-x)$$

  → exactly as from handbag diagram at quark level

Duality in electron-nucleon scattering

"Bloom-Gilman duality"



Bloom, Gilman
PRL ***85****, 1185 (1970)*

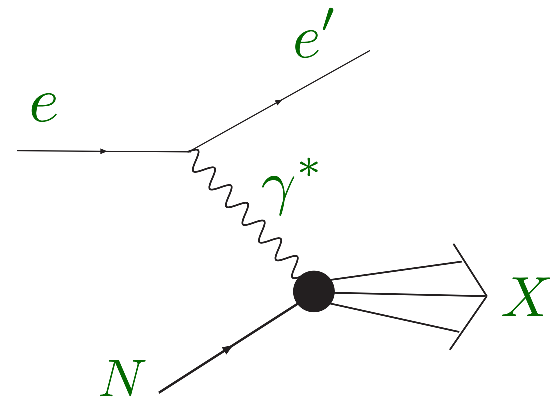
$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

"hadrons" "quarks"

Electron-nucleon scattering

- Inclusive cross section for $eN \to eX$



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$

$$\left.\begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^{\,2} - \nu^2 = 4EE'\sin^2\frac{\theta}{2} \end{aligned}\right\} \quad x = \frac{Q^2}{2M\nu}$$

Bjorken scaling variable

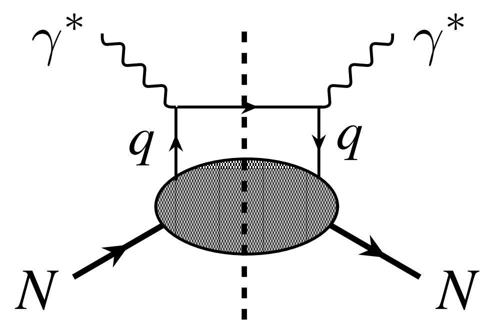
- F_1 , F_2 structure functions

⟶ contain all information about structure of nucleon

- In deep inelastic region ($W \gtrsim 2$ GeV, $Q^2 \gtrsim 1$ GeV^2), structure function given by quark and antiquark ("parton") distributions

$$F_2(x,Q^2) = x \sum_q e_q^2 \, q(x,Q^2)$$
$$= \frac{4}{9}x(u+\bar{u}) + \frac{1}{9}x(d+\bar{d}) + \frac{1}{9}x(s+\bar{s}) + \cdots$$

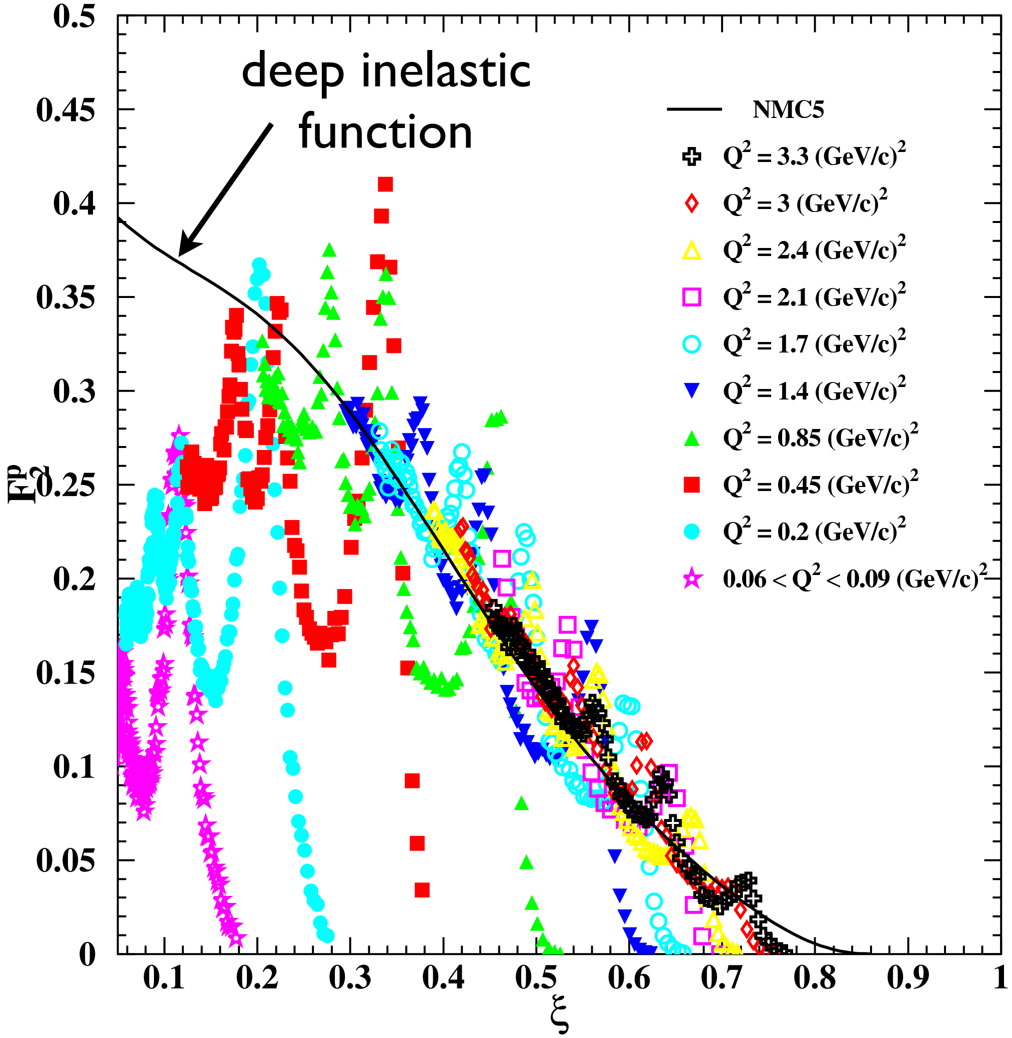
→ $q(x,Q^2)$ = probability to find quark type "q" in nucleon, carrying momentum fraction x



- In resonance region ($W \lesssim 2$ GeV), or at low Q^2 ($Q^2 \lesssim 1$ GeV^2) can no longer resolve individual quark structure

→ see *gross features* of hadron (complex, multi-parton effects)

Duality in electron-nucleon scattering



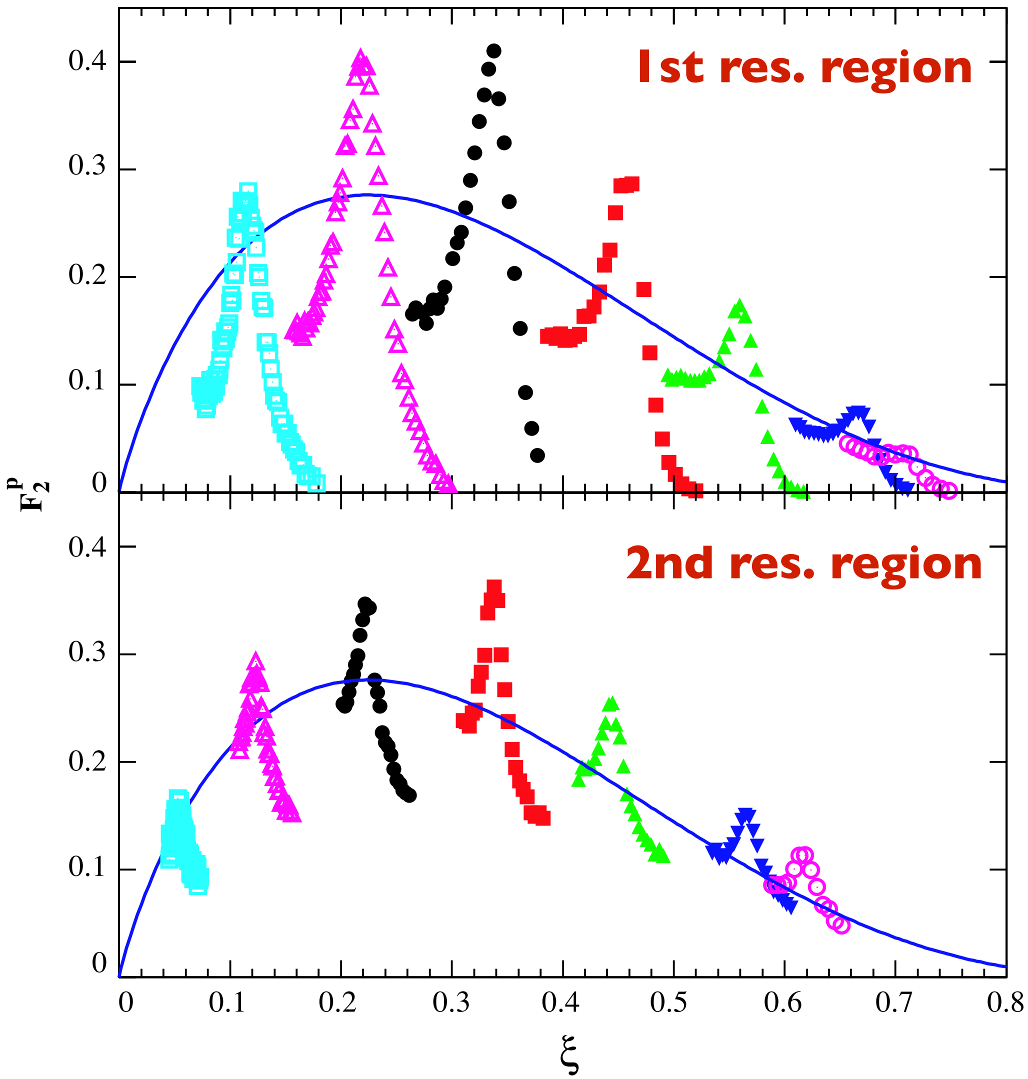
average over
(strongly Q^2 dependent)
resonances
$\approx Q^2$ independent
scaling function

"Nachtmann" scaling variable

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

Niculescu et al., PRL ***85****, 1182 (2000)*
WM, Ent, Keppel, PRep. ***406****, 127 (2005)*

Duality in electron-nucleon scattering



→ also exists *locally* in individual resonance regions

Duality and QCD

- Operator product expansion

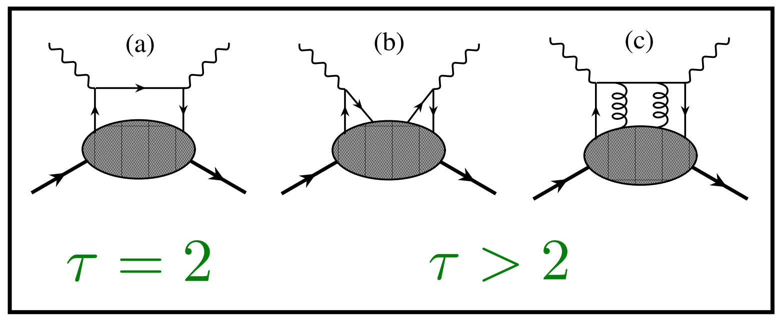
→ expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$



Duality and QCD

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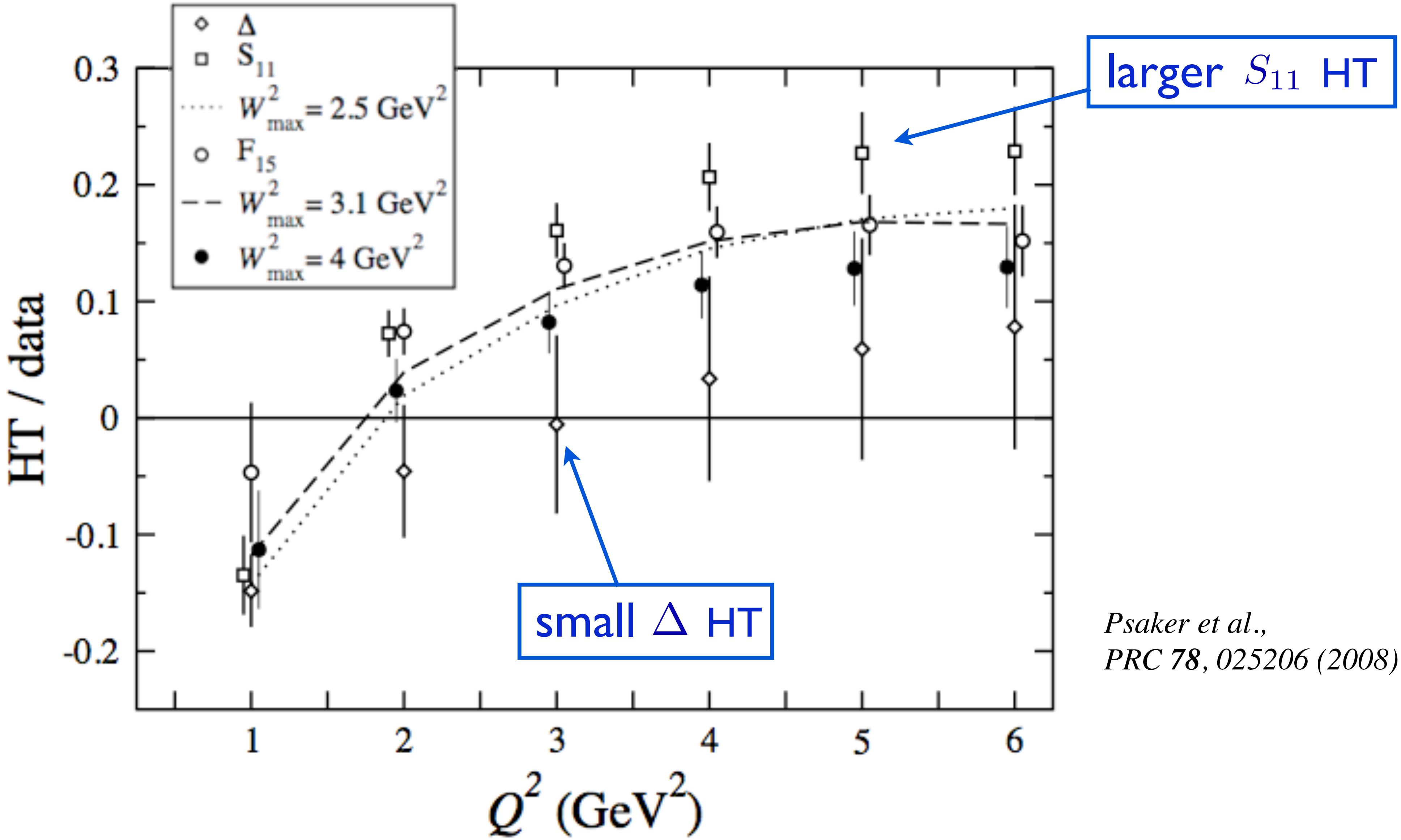
de Rujula, Georgi, Politzer
Ann. Phys. ***103****, 315 (1975)*

- If moment $\approx$ independent of Q^2

  $\longrightarrow$ higher twist terms $A_n^{(\tau>2)}$ small

- Duality $\longleftrightarrow$ suppression of higher twists

Analysis of JLab F_2^p resonance region data



Psaker et al.,
PRC ***78****, 025206 (2008)*

→ higher twists $< 10-15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$

Resonances & twists

- Total higher twist "*small*" at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$
- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)

  $\longrightarrow$ nontrivial interference between resonances

Resonances & twists

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- Can we understand this dynamically, at quark level?
  $\longrightarrow$ is duality an accident?
- Can we use resonance region data to learn about *leading twist* structure functions?
  $\longrightarrow$ expanded data set has potentially significant implications for global PDF studies

- Consider simple quark model with spin-flavor symmetric wave function

form factors

→ *coherent* scattering from quarks $d\sigma \sim \left(\sum_i e_i\right)^2$

structure functions

→ *incoherent* scattering from quarks $d\sigma \sim \sum_i e_i^2$

- For duality to work, these must be equal

→ how can *square of a sum* become *sum of squares*?

Dynamical cancellations

→ *e.g.* for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

$$F(\nu, \mathbf{q}^2) \ \sim \ \sum_n \left| G_{0,n}(\mathbf{q}^2) \right|^2 \ \delta(E_n - E_0 - \nu)$$

→ charge operator $\Sigma_i \ e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites *even* partial waves with strength $\propto (e_1 + e_2)^2$ *odd* partial waves with strength $\propto (e_1 - e_2)^2$

→ resulting structure function

$$F(\nu, \mathbf{q}^2) \ \sim \ \sum_n \left\{ (e_1 + e_2)^2 \ G^2_{0,2n} + (e_1 - e_2)^2 \ G^2_{0,2n+1} \right\}$$

→ if states degenerate, *cross terms* $(\sim e_1 e_2)$ *cancel* when averaged over nearby *even and odd parity* states

Close, Isgur, PLB ***509****, 81 (2001)*

Dynamical cancellations

→ duality is realized by summing over at least one complete set of *even* and *odd* parity resonances

→ in NR Quark Model, even & odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

$\lambda\ (\rho) =$ (anti) symmetric component of ground state wave function

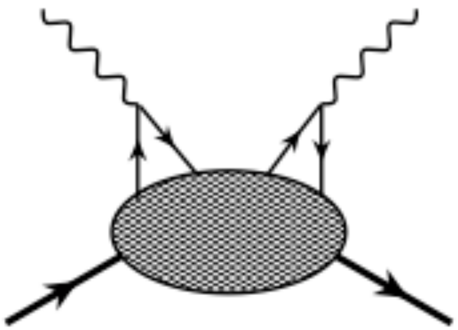
→ summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

$$\frac{F_1^n}{F_1^p} = \frac{18}{27} = \frac{2}{3}$$ as in parton model (if u=$2d$) !

→ similar realizations of duality seen in other models

*Close, WM, PRC **68**, 035210 (2003)*

Accidental cancellations of charges?



<u>cat's ears diagram</u> (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\sum_i e_i\Big)^2 - \sum_i e_i^2$$

↑ *coherent* ↑ *incoherent*

proton $\quad \mathrm{HT} \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0 \; !$

neutron $\quad \mathrm{HT} \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0$

Brodsky
hep-ph/0006310

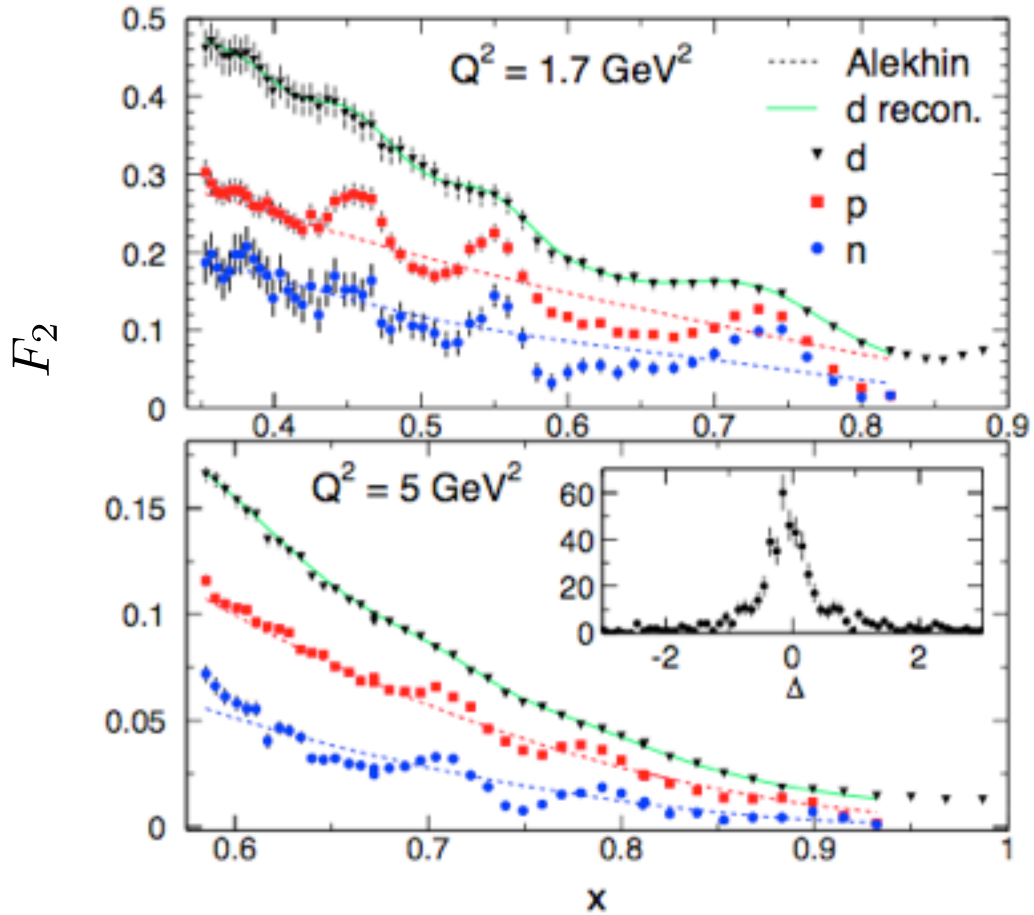
→ duality in proton a *coincidence!*

→ should *not* hold for neutron

Neutron: the smoking gun

- Duality in *neutron* more difficult to test because of absence of free neutron targets

- New extraction method (using iterative procedure for solving integral convolution equations) has allowed first determination of F_2^n in resonance region & test of neutron duality



Malace, Kahn, WM, Keppel
PRL ***104****, 102001 (2010)*

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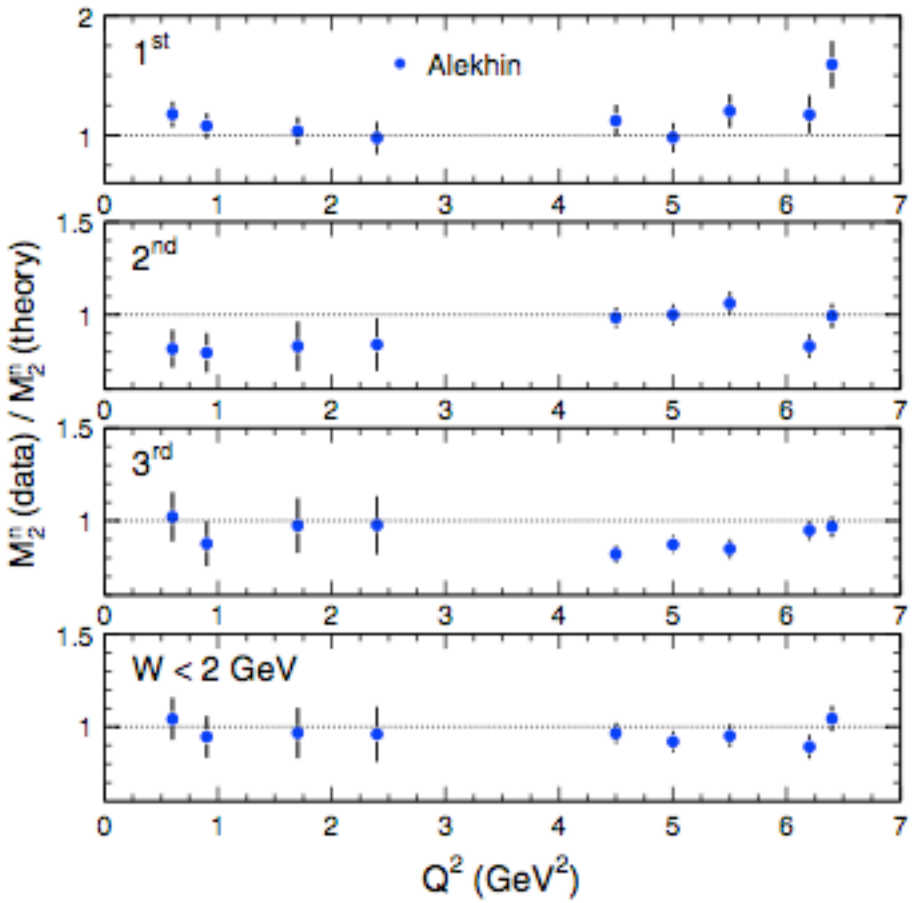


→ "theory": fit to $W > 2$ GeV data

Alekhin et al., 0908.2762 [hep-ph]

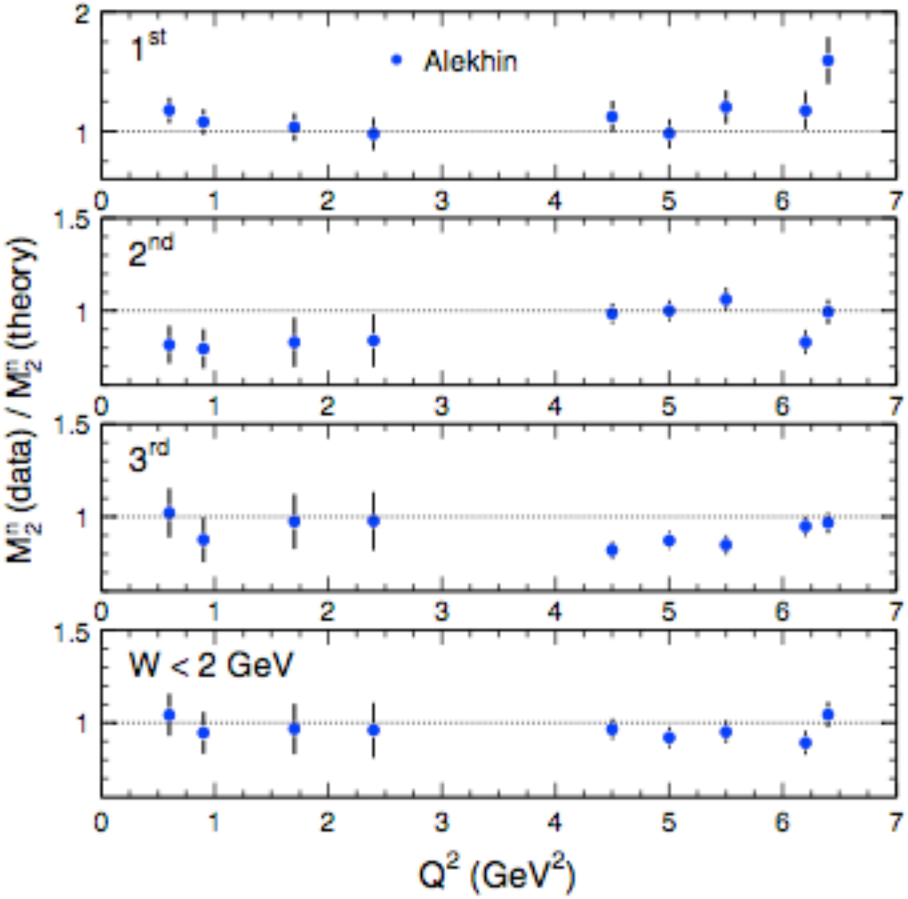
→ *locally*, violations of duality in resonance regions < 15–20% (largest in Δ region)

→ *globally*, violations < 10%

Malace, Kahn, WM, Keppel
*PRL **104**, 102001 (2010)*

⇨ duality is *<u>not</u> accidental*, but a general feature of resonance–scaling transition!

Neutron: the smoking gun



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⇒ use resonance region data to learn about *leading twist* structure functions?

CTEQ-JLab (CJ) global PDF analysis *

- New global NLO analysis of expanded set of p and d data (DIS, pp, pd) including large-x, low-Q^2 region

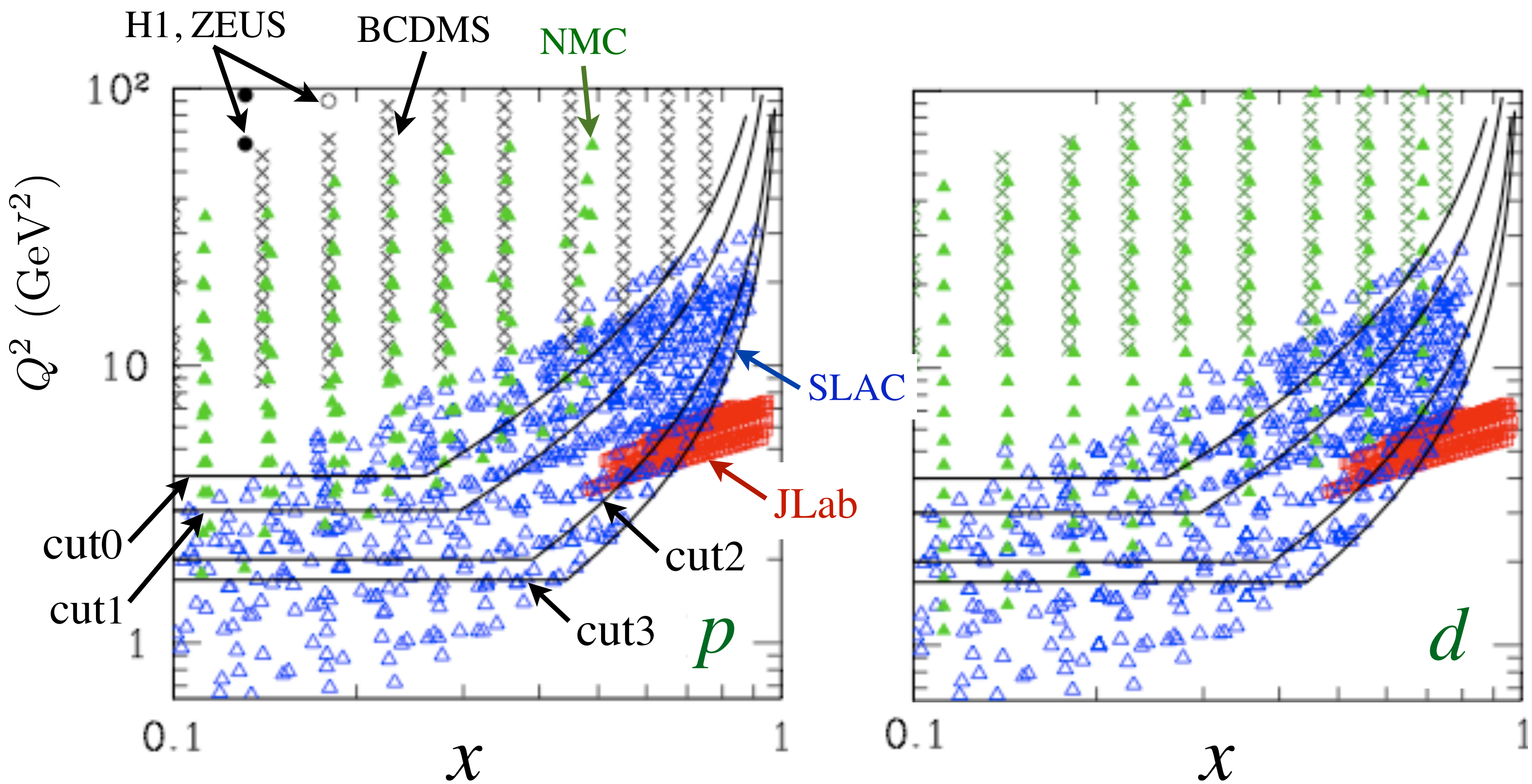
- Systematically study effects of Q^2 & W cuts
  → down to $Q \sim m_c$ and $W \sim 1.7$ GeV

- Correct for *nuclear* effects in the deuteron, subleading $1/Q^2$ corrections (target mass, higher-twists)

- Dependence on choice of PDF parametrization

* CJ collaboration: A. Accardi, J. Owens, WM (theory)
E. Christy, C. Keppel, P. Monaghan, L. Zhu (expt.)
http://www.jlab.org/CJ/

CJ kinematic cuts



cut0: $Q^2 > 4$ GeV^2, $W^2 > 12.25$ GeV^2

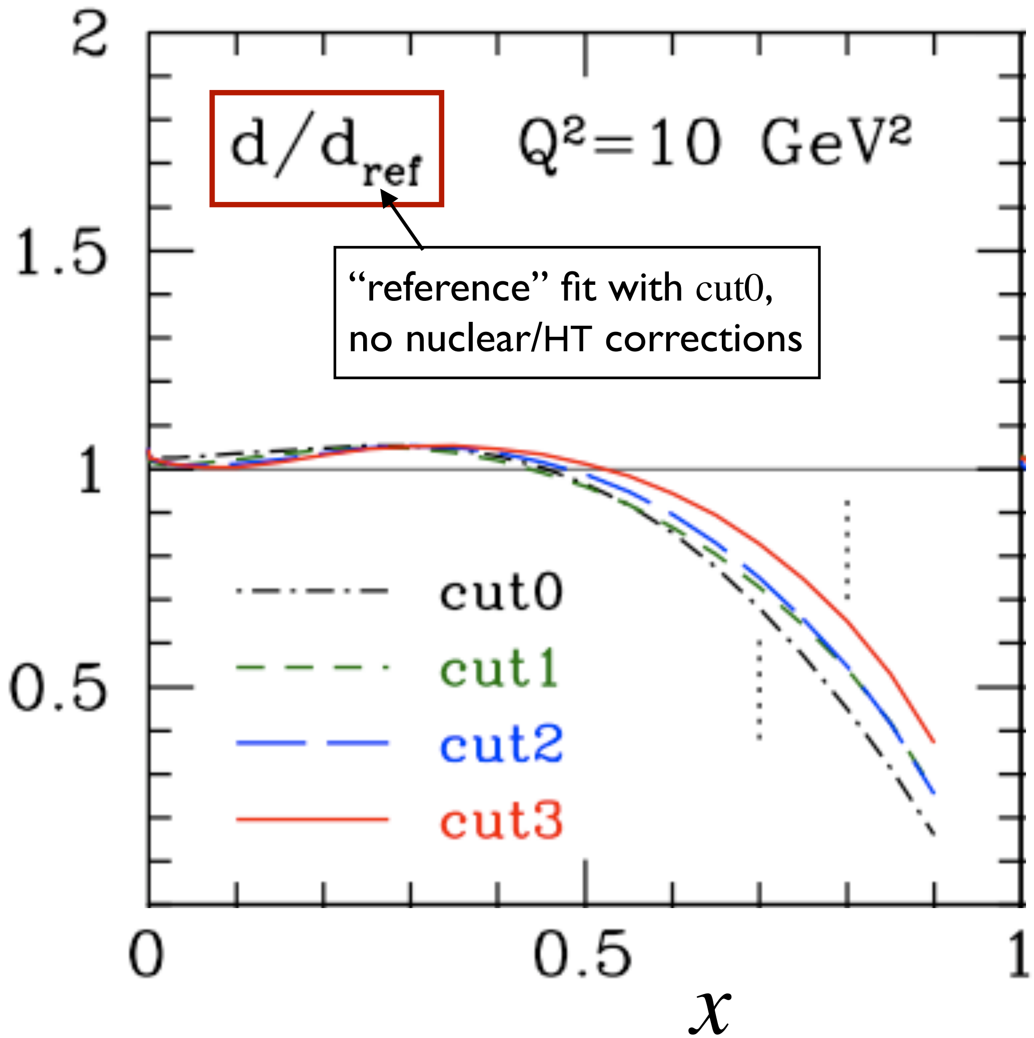
cut1: $Q^2 > 3$ GeV^2, $W^2 > 8$ GeV^2

cut2: $Q^2 > 2$ GeV^2, $W^2 > 4$ GeV^2

cut3: $Q^2 > m_c^2$, $W^2 > 3$ GeV^2

factor 2 increase in DIS data from cut0 $\longrightarrow$ cut3

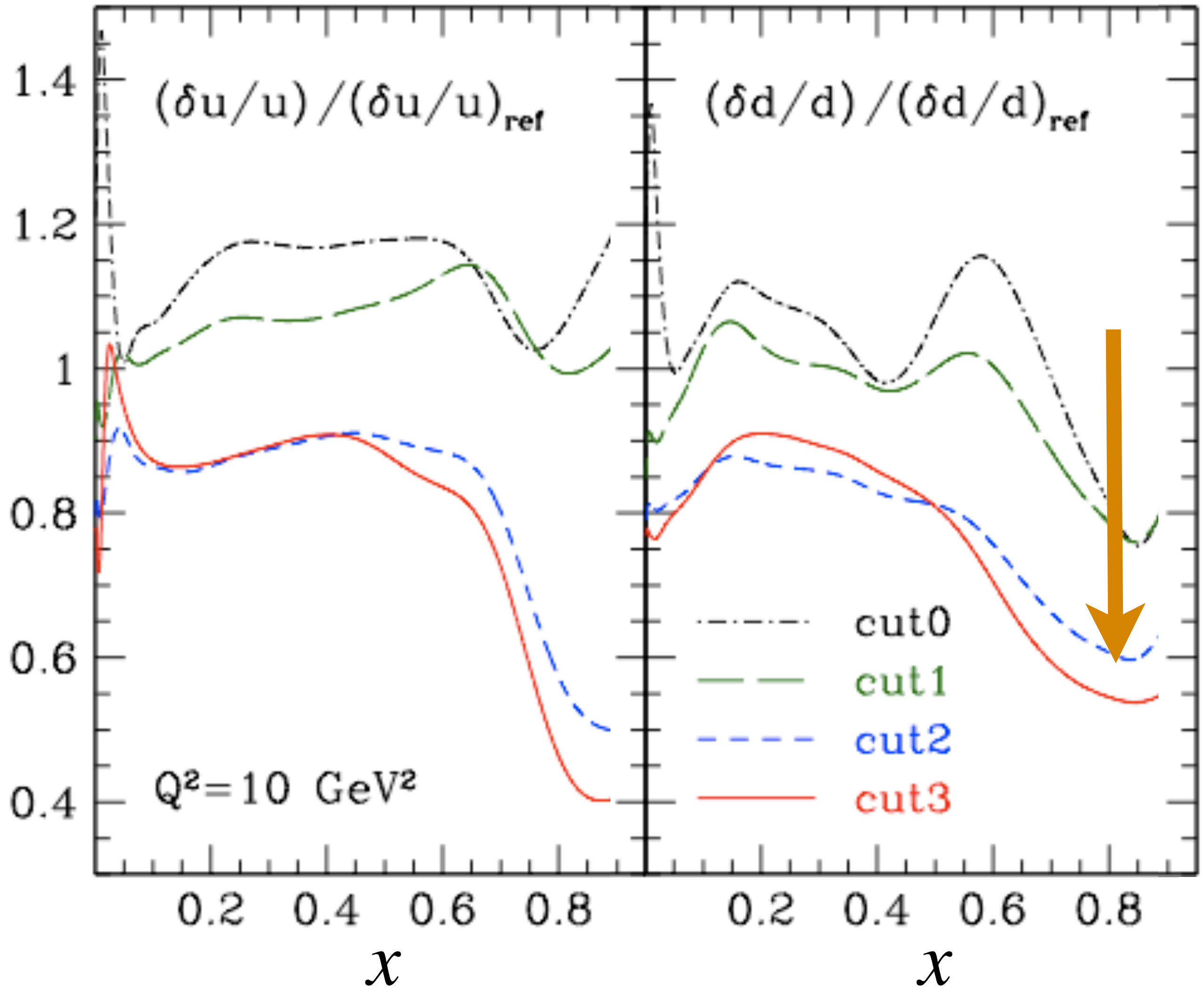
- PDFs remarkably *stable* with respect to cut reduction, as long as finite-Q^2 corrections included



Accardi et al.
PRD ***81****, 034016 (2010)*

→ d quark behavior driven by nuclear corrections at high x

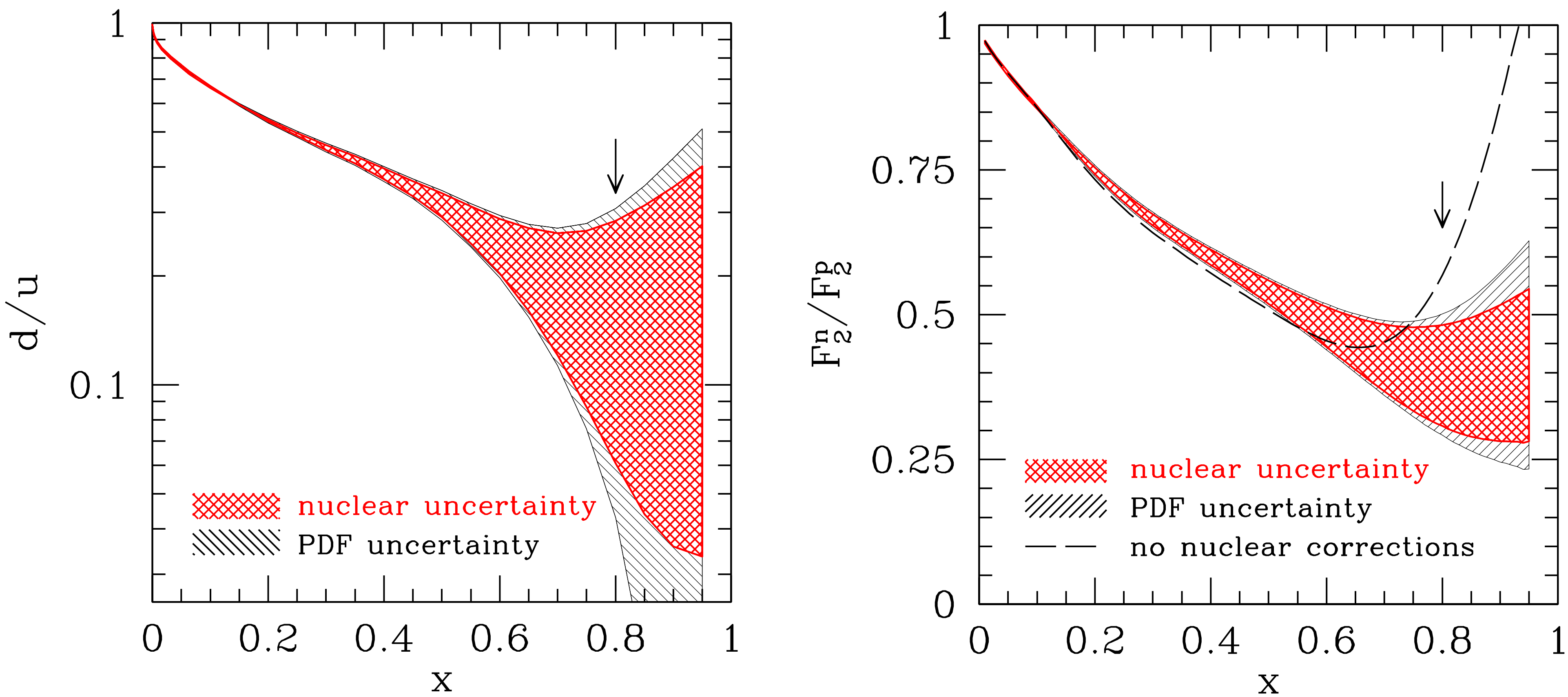
- Larger database with weaker cuts leads to significantly reduced errors, especially at large x



Accardi et al.
PRD ***81****, 034016 (2010)*

→ up to 40–60% error reduction when cuts extended into resonance region

- Vital for large-x analysis, which currently suffers from large uncertainties (mostly due to nuclear corrections)



*Accardi et al., PRD **84**, 014008 (2011)*

→ uncertainty in d feeds into larger uncertainty in g at high x (important for LHC physics!)

Brady et al., arXiv:1110.5398

Summary

- Remarkable confirmation of quark-hadron duality in *proton* and *neutron* structure functions

  → duality-violating higher twists ~ 10–15% in few-GeV range

- Confirmation of duality in *neutron* suggests origin in dynamical cancellations of higher twists

  → duality *not* due to accidental cancellations of quark charges

- Practical application of duality

  → use resonance region data to constrain *leading twist* PDFs (global PDF analysis underway)

  → stable fits at low Q^2 and large x with significantly reduced uncertainties

The End

GAUSTEQ
Germany and U.S. Nuclear Theory Exchange Program for QCD Studies of Hadrons and Nuclei

- Newly approved DOE program for US–Germany exchange in hadron/nuclear theory, centered around JLab and GSI–FAIR

- Fully funds US-based physicists for up to 2–4 week collaborative visits to Germany

- See http://www.jlab.org/GAUSTEQ/
or contact one of the PIs (Jo Dudek, WM, Christian Weiss)
through <gausteq@jlab.org>